Part 1: Jointly Continuous (100 points)

Philip is struggling to come up with a good example of a joint distribution function to use as an example in class. Finally, he comes up with the following:

$$f\left(x,y\right)=Cx^{2}y^{2} 0<y<1, y^{3}<x<y^{1/3}$$

Please make sure that if you give decimal approximations (which would be nice for grading), they have enough significant digits to show all the relevant pieces of information.

1. (10 points) In order for this to be a valid PDF, what is *C*?
2. (10 points) What is $f\_{X}\left(x\right)$?
3. (10 points) What is $E\left[X\right]?$
4. (10 points) What is the second moment of *X*?
5. (10 points) What is the variance of *X*?
6. (10 points) What is $E\left[XY\right]?$
7. (10 points) What is the Covariance of *X* and *Y*?
8. (10 points) What is $f\_{X|Y}\left(y\right)?$
9. (10 points) What is $P\left\{0<x<\frac{1}{4}\right\}?$
10. (10 points) What is $P\left\{y=\frac{1}{8}\right\}$?

Part 2: Discrete Variables (96 points)

Let X be a discrete random variable with the following density function:

$$f\_{X}\left(n\right)=\left\{\begin{array}{c}ce^{-2}\frac{2^{n}}{n!} n\geq 0\\c3^{n} n<0\end{array}\right.$$

1. (9 points) In order for this to be a valid PDF, what is *C*?
2. (7 points) What is $P\left(X<1\right)$?

Define the variables *U, V* as follows:

$$U=\left\{\begin{array}{c}-X X<0\\0 X\geq 0\end{array}\right. , V=\left\{\begin{array}{c}0 X<0\\X X\geq 0\end{array}\right.$$

1. (12 points) What are the density functions of *U* and *V*$?$

Let R be a geometric random variable with density function $f\_{R}\left(r\right)= 2^{-r}$ for $r=1,2,…$ that is independent of U.

1. (14 points) What is $P\left(U\leq R\right)$?
2. (15 points) Define $W\_{1}=U+V$ and $W\_{2}=U-V.$ What is $Cov(W\_{1},W\_{2})$?

Let *T* be a bernoulli random variables that takes value 1 with probability 0.7. Assume *T* is independent of both *U, V*.

1. (13 points) What is $E\left(U∙T+V∙\left(1-T\right)\right)?$

Let $V\_{1}, V\_{2}, V\_{3}, …$ be a sequence of independent and identically distributed random variables each having the same distribution as $V$.

1. (14 points) Use the central limit theorem to approximately find $P(20<\sum\_{i=1}^{20}V\_{i}<30)$.
2. (12 points) Let $N$ be a poisson random variable with mean 5. Assume *N* is independent of the $V\_{i}$’s. Define $Z=\sum\_{i=1}^{N}V\_{i}$. What is $E\left(Z\right)?$